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GLBR 22
MATHEMATICS

CONTENTS

1. ROOTS OF A QUADRATIC: \( AX^2 + BX + C = 0 \)
   Computes the roots of any quadratic equation with real coefficients.

2. ROOTS OF A POLYNOMIAL: \( P(X) = A_0 + A_1 X + \ldots + A_n X^n \)
   Finds the real and complex roots of an nth degree polynomial with real coefficients by Bairstow's algorithm.

3. HALF-INTERVAL SEARCH FOR ROOTS
   Given a function, \( f(x) \), and an interval, this program searches the interval for 2 points with opposite signs. If 2 points with opposite signs are found, a root between is calculated by the half-interval search method.

4. REAL ROOTS OF A POLYNOMIAL: \( A_1 X^N + A_2 X^{N-1} + \ldots + A_N X + A_{N+1} = 0 \)
   Computes, by the Newton-Raphson method, roots of a polynomial with real coefficients of degree \( N(N < 100) \) given estimated values of the roots.

5. SIMPSON'S RULE: \( \int_a^b f(x) = \frac{D}{3} (f(A) + 4f(A+D) + 2f(A+2D) + \ldots + 2f(B-2D) + 4f(B-D) + f(B)) \)
   This program uses Simpson's rule to calculate the definite integral of a known function or of a continuous function for which we have determined empirically a number of equally spaced values.

6. NUMERICAL INTEGRATION (Romberg's Method)
   Computes the definite integral of a function to 4 significant digits using the trapezoidal rule with Romberg's extrapolation method.

7. RUNGE-KUTTA
   This program solves a system of \( N(N \leq 10) \) first-order differential equations with initial conditions using a Runge-Kutta 4th-order method with Runge's coefficients.

8. GAUSSIAN QUADRATURE (20-point)
   Approximates the definite integral of a function over a given interval by dividing the interval into a specified number of equal subintervals and summing the results of 20-point Gaussian quadratures done over each subinterval.
9. DERIVATIVE (Difference Quotients)

Gives a good approximation to the derivative of a function, \( f(x) \), at the point \( x=a \), and shows successive difference quotients for values of \( x \) approaching \( a \) from the right.

10. MATRIX INVERSION (Gauss-Jordan Elimination Method)

Inverts an \( N \times N \) matrix with real elements by the Gauss-Jordan elimination method.

11. MATRIX INVERSION (Gauss-Jordan Done In Place)

Inverts a real \( N \times N \) matrix by a modified Gauss-Jordan elimination method done in place.

12. EIGENVALUE AND EIGENVECTOR

Calculates the eigenvalues and eigenvectors of a real symmetric matrix of dimension \( N \) by a modified Jacobi method.

13. VECTOR OPERATIONS

Performs vector addition, vector subtraction, dot product, and cross product of 2 vectors in 3-space.

14. VECTOR ANALYSIS

Computes the magnitude of vectors, the angle between vector and axes, and the angle between 2 vectors.

15. SOLUTION OF SIMULTANEOUS EQUATIONS (Gauss-Jordan)

Solves \( N \) linear equations in \( N \) unknowns. The system of equations is put into the following form: \( AX=B \) where \( A= \) coefficient matrix, \( B= \) column matrix of constants, \( X= \) column vector of unknowns.

16. MATRIX ADDITION, SUBTRACTION, AND SCALAR MULTIPLICATION

Adds an \( N \times M \) matrix \( B \) to an \( N \times M \) matrix \( A \) or subtracts matrix \( B \) from matrix \( A \) or multiplies matrix \( A \) by a scalar, \( k \). (\( N, M \leq 10 \))

17. MATRIX MULTIPLICATION

Multiplies the \( N \times M \) matrix \( A \) on the right by the \( M \times P \) matrix \( B \).

18. SOLUTION OF SIMULTANEOUS EQUATIONS (Gauss-Seidel)

Solves \( N \) (\( N \leq 8 \)) simultaneous equations in \( N \) unknowns by the Gauss-Seidel iterative method.
19. LINEAR PROGRAMMING
Solves a linear programming problem by the simplex method.

20. COMPLEX DETERMINANT
Evaluates the determinant of a matrix with complex elements by the Crout method. The matrix must be square.

21. HYPERBOLIC FUNCTIONS AND INVERSE HYPERBOLICS
Computes hyperbolics and inverse hyperbolics for angles in radians.

22. SIN, COS, TAN, SINH, COSH, TANH - COMPLEX ARGUMENT
Computes the values of the Sin(z), Cos(z), Tan(z), Sinh(z), Cosh(z), Tanh(z) where z is a complex number of the form A+Bi.

23. ANGLE CONVERSION I
Converts an angle given in radians to degrees, minutes, and seconds.

24. ANGLE CONVERSION II
Converts an angle from degrees, minutes, seconds to radians.

25. TRIGONOMETRIC POLYNOMIAL
Calculates the value of a trigonometric polynomial of the form: \( f(x) = A_1 \sin(x) + B_1 \cos(x) + \ldots + A_N \sin(Nx) + B_N \cos(Nx) \).

26. PLANE TRIANGLE SOLUTION
Calculates the unknown sides and angles of a triangle given one side and any 2 other parts.

27. COORDINATE CHANGE
Converts cartesian coordinates \((X, Y)\) to polar coordinates \((R, A)\) or polar coordinates \((R, A)\) to cartesian coordinates \((X, Y)\).

28. AREA OF RECTILINEAR SURFACE
Calculates the area of a plane surface enclosed by line segments when the coordinates of the vertices are given.

29. LINEAR INTERPOLATION
Linearly interpolates points on a line given 2 other points on the line, \((X_1, Y_1), (X_2, Y_2)\).
30. LAGRANGIAN INTERPOLATION

Given N(N<70) distinct data points, the Y-coordinates corresponding to inputed X-values are computed by the Lagrangian interpolation formula for unequally spaced data.

31. GREATEST COMMON DIVISOR OF 2 INTEGERS

Computes the greatest common divisor of 2 integers by the Euclidean algorithm.

32. PRIME FACTORIZATION OF INTEGERS

Factors any integer into a product of prime numbers.

33. PERMUTATIONS AND COMBINATIONS

Computes the number of permutations and the number of combinations of N items taken R at a time.

34. LOG B TO BASE A

Computes Log B to Base A by: \( \log_A(B) = \log_e(B) / \log_e(A) \).

35. SECOND DEGREE EQUATION I

Converts the explicit form of a second degree equation to the general form.

36. EXPLICIT SECOND DEGREE EQUATION

Converts an explicit second degree equation from \( Y=f(x) \) to \( X=g(y) \).

37. SECOND DEGREE EQUATION II

Converts a general second degree equation to its explicit form.

38. ALGEBRA OF COMPLEX NUMBERS

This program performs any sequence of the operations +, -, *, / on complex numbers in the form A+Bi.

39. HYPERGEOMETRIC FUNCTION

Evaluates a hypergeometric function of the form:

\[
M(Z) = \frac{AZ}{B} + \frac{A(A+1)Z^2}{B(B+1)2!} + \ldots + \frac{A(A+1)\ldots(A+N-1)Z^N}{B(B+1)\ldots(B+N-1)N!}
\]

40. SQUARE ROOT OF A COMPLEX NUMBER

Computes the square root of a complex number A+Bi.
41. BESSEL FUNCTION

Calculates values of the Bessel function $J_I(x)$, $I$ is an integer. ($I, X > 0$)

42. GAMMA FUNCTION

Evaluates the gamma function, $\Gamma(x)$, by a polynomial approximation. ($-50 < X < 50$)

43. FOURIER ANALYSIS (Defined Function)

Calculates the Fourier coefficients of a given $2\pi$ periodic function on a discrete set of $2N+1$ equally spaced $X$-values. The maximum harmonic, $N$, to be calculated must be $\leq N$.

44. FOURIER ANALYSIS (Tabulated Function)

Calculates the Fourier coefficients of a tabulated function in the range $0$ to $2\pi$. It is assumed the interval is divided into $2N+1$ subintervals and $2N+1$ observations have been made. The maximum harmonic, $N$, to be calculated must be $\leq N$. 


1. LINEAR REGRESSION: \( Y = A + Bx \)

Fits the curve \( y = A + Bx \) to a set of \( N \) data points by the method of least squares. Also, an analysis of regression is performed -- the regression table, F-value, coefficient of determination, coefficient of correlation, and standard error of estimate are printed out (or displayed). The user may estimate values of \( y \) from the regression curve by inputing values of \( x \).

2. MULTIPLE LINEAR REGRESSION

This is a two-part program. The first segment fits the curve:

\[
y = b_0 + b_1x_1 + b_2x_2 + \ldots + b_Mx_M
\]

to a set of \( N \) data points by the method of least squares. The second segment performs an analysis of regression -- the regression table, F-value, coefficient of determination, coefficient of multiple correlation, and standard error of estimate are printed out (or displayed). The user may also estimate values of \( y \) from the regression curve by supplying values for the independent variables \( (x_1, x_2, \ldots, x_M) \).

3. Mth ORDER LINEAR REGRESSION

This is a two-part program. The first segment fits the curve:

\[
y = b_0 + b_1x + b_2x^2 + \ldots + b_Mx^M
\]

to a set of \( N \) \((x, y)\)--data points by the method of least squares. The second segment performs an analysis of regression -- the regression table, F-value, coefficient of determination, coefficient of correlation, and standard error of estimate are printed out (or displayed). The user may also estimate values of \( y \) from the regression curve by supplying values for \( x \).

4. EXPONENTIAL REGRESSION

Fits the curve \( Y = Ae^{BX} \) to a set of \( N \) data points by the method of least squares. An analysis of regression is done on the linear regression -- the regression table, F-value, coefficient of determination, coefficient of correlation, and standard error of estimate are printed out (or displayed). The user may estimate values of \( Y \) from the exponential regression curve by inputing values of \( X \).
5. GEOMETRIC REGRESSION

Fits the curve $Y = Ax^B$ to a set of $N$ data points by the method of least squares. An analysis of regression is done on the linear regression table, F-value, coefficient of determination, coefficient of correlation, and standard error of estimate are printed out (or displayed). The user may also estimate values of $Y$ from the geometric regression curve by inputting values of $X$.

6. LINEAR CORRELATION

Computes the coefficient of linear correlation, $R$, between 2 variables, $X$ and $Y$.

7. CORRELATION MATRIX

Given a set of $N$ observations in $M$ variables, the correlation coefficient between each pair of variables is computed.

8. ONE-WAY ANAYLSIS OF VARIANCE

Performs a 1-way analysis of variance on up to 99 groups of data. The analysis of variance table and F-value are printed out (or displayed).

9. TWO-WAY ANAYLSIS OF VARIANCE

Computes the analysis of variance table and the F-values for the row and column variance in a two-factor experiment.

10. ANALYSIS OF VARIANCE - LATIN SQUARES

Computes the analysis of variance table and the F-values for column, row, and treatment variance for a simple Latin square design. The user must supply the matrix of treatment assignments and the matrix of data.

11. CHI-SQUARE TEST AND DISTRIBUTION

The Chi-Square ($\chi^2$) test compares an observed distribution with an assumed distribution.

12. CHI-SQUARE ANALYSIS

Computes, for an $N\times M$ contingency table, the value of Chi-Square, the number of degrees of freedom, the expected value for each cell and the Chi-Square contribution from each cell.
13. **t-TEST**

Calculates the $t$-statistic to test whether or not two samples have the same population means. The test is performed for one of four hypotheses.

14. **WILCOXON MATCHED-PAIRS SIGNED-RANKS TEST**

Performs the Wilcoxon matched-pairs signed-ranks test on a set of $N$ pairs of data. Note: it is best to leave tie scores for a given pair out of the analysis.

15. **MANN-WHITNEY U TEST**

Performs the Mann-Whitney U-test given two samples.

16. **NORMAL FREQUENCY AND DISTRIBUTION FUNCTIONS**

Computes values of the normal frequency and normal distribution functions.

17. **NEGATIVE BINOMIAL DISTRIBUTION**

The negative binomial distribution given by:

$$ P(K, R, P) = \frac{(R + K - 1)!}{K! \cdot (R - 1)!} \cdot p^R \cdot q^K $$

is a discrete distribution used in solving waiting time problems. It calculates the probability that the $R$th success will occur at a given trial number $R + K$, in a succession of $N$ Bernoulli trials. This program is restricted to $R$ a positive integer.

18. **BINOMIAL DISTRIBUTION**

The binomial distribution given by:

$$ P(K, N, P) = \frac{N!}{K! \cdot (N - K)!} \cdot p^K \cdot q^{N-K} $$

is a discrete distribution giving the probability of obtaining exactly $K$ successes in $N$ Bernoulli trials.

19. **POISSON DISTRIBUTION**

The Poisson distribution given by:

$$ P(K, \lambda) = \frac{e^{-\lambda} \cdot \lambda^K}{K!} $$

is a discrete distribution concerned with the occurrence of relatively rare events. $K$ is the frequency, $P(K, \lambda)$ is the probability associated with that frequency, and $\lambda$ is the expected frequency.
20. F-VALUE

Computes the probability of an F-ratio with N degrees of freedom in the numerator and D degrees of freedom in the denominator.

21. t-VALUE

Computes the probability of a t-value for a two-tailed test with N degrees of freedom.

22. RANDOM NORMAL DEVIATES

Generates random normal deviates with a mean of zero and a variance of one.

23. MEAN, VARIANCE, STANDARD DEVIATION I

Computes the mean, variance, and standard deviation for ungrouped data, taken from either a population or a sample.

24. MEAN, VARIANCE, STANDARD DEVIATION II

Computes the mean, variance, and standard deviation for grouped data, taken from either a population or a sample.

25. GEOMETRIC MEAN AND STANDARD DEVIATION

Computes the geometric mean and geometric standard deviation for a geometrically normal set of data.

26. CROSS-COVARIANCE OF TIME SERIES

Finds the cross-covariances of series A with series B (which leads and lags A). The lag covariance and lead covariance are calculated for lags and leads of 0, 1, 2, ..., L-1 where L ≤ N and N = number of elements in series A and B.

27. AUTOCOVARIANCE OF A TIME SERIES

Finds the autocovariances of a time series for lags of 0 to L where L ≤ N and N = number of elements in the series. (N ≤ 96.)

28. SYSTEM RELIABILITY

Calculates the system reliability when chance failure is present along with wearout.

29. ERROR FUNCTION

Computes the definite integral of the function

\[ E(X) = \frac{2}{\sqrt{\pi}} e^{-x^2} \]

between the limits of 0 and X, using the trapezoidal rule with Romberg's extrapolation. The integral is calculated to 4 significant digits.
30. TALBOT'S FORMULA

Estimates the area of waterway opening required for culverts.

31. MANNING'S FORMULA

Determines the discharge quantity for open-channel flow by combining Manning's Formula for velocity of water flow with the general flow formula.

32. HEADLOSS IN A PIPE

Calculates the headloss between two reservoirs on different levels connected by a pipe.

33. BERNOULLI'S EQUATION

This program uses Bernoulli's equation to compute the headwater depth of culverts flowing full.

34. WARPING STRESS DUE TO TEMPERATURE DIFFERENTIAL

Computes the warping stress in 2 directions which exist in the center of a slab of concrete pavement.

35. PRESSURE DUE TO SURFACE LOADS, POINT LOADS, FINITE OR INFINITE LINE LOADS

Computes the horizontal unit pressure due to a point load, at any point on the wall of a backfill surface; or the unit lateral pressure, due to a finite or infinite line load, at any depth opposite one end of a parallel strip load on the backfill.

36. BEAM

This program recommends steel beams to use for a number of common applications.

37. OIL WELL DEPLETION

This program calculates the number of years that an oil well will produce given the current production rate, the minimum number of barrels of oil that must be produced to cover expenses and the reserve to be recovered on decline. The cumulative production and the unit production for each year is also computed.

38. NETWORK IMPEDANCE, FINDING A SERIES OR PARALLEL CIRCUIT

This program is designed to parallel a circuit that is in series (or vice-versa) by the leaning ladder method.
39. CHARACTERISTIC GENERATOR RESISTANCE AND SOURCE EMF VOLTAGE

Computes the characteristic generator resistance and source emf voltage of an efficient rf switched amplifier whose output power swings with mismatch.

40. "ERLANG B" EQUATION

In the administration of a telephone or telex exchange, it is common to estimate the load upon groups of equipments by reading Erlang meters associated with each grouping of equipments. The purpose of such activities is to determine the grade of service given the number of equipments and the traffic offered.
1. NUMBER OF SEMI-ANNUAL PERIODS BETWEEN TWO DATES (360 DAY-YEAR)

Calculates the number of semi-annual periods between 2 dates, based on a 360 day-year.

2. BOND DOLLAR PRICE

Given the Settlement and Maturity dates, Coupon, Yield and an increment value for the Yield, the program calculates a table showing the dollar price at the different Yield values. Maturity must be greater than or equal to six months.

3. BOND YIELD (BASIS)

Given the Settlement and Maturity Dates, Coupon, Dollar Price and an increment value for the Dollar Price, the program calculates a table showing the Yield at the different Dollar Price values. Maturity must be greater than or equal to six months.

4. DISCOUNT AND PRICE ON DISCOUNT COMMERCIAL PAPER

Calculates the discount and net cost on discount commercial paper.

5. INTEREST BEARING COMMERCIAL PAPER

Calculates the interest rate and accrued interest at maturity on interest bearing commercial paper.

6. NUMBER OF DAYS BETWEEN TWO DATES

Calculates the actual number of days between two dates.

7. MORTGAGE PAYMENT

Computes the monthly payment and displays amortization table.

8. DAY OF YEAR

Computes the day of the week that a given date falls on.

9. ANNUITY

Calculates the accumulated amount of an annuity.

10. ANNUAL DEBT PAYMENT

Calculates the annual debt payment given: annual interest rate, life, salvage value, and first cost.
11. PRESENT INVESTMENT

Finds the present amount necessary to provide a desired sum after \(N\) years.

12. NOMINAL INTEREST RATE

Finds the nominal interest rate for a given principal invested for \(N\) years compounded \(M\) times/year amounting to sum \(S\).

13. EFFECTIVE INTEREST RATE

Finds the effective interest rate for a principal invested for \(N\) years amounting to sum \(S\).

14. INVESTMENT WITHDRAWAL

Finds the amount that can be withdrawn from an initial investment \(M\) times/year for \(N\) years, leaving nothing at end, if the interest rate is \(i\).

15. INITIAL INVESTMENT

Finds the investment necessary to enable one to withdraw a given amount, \(M\) times/year for \(N\) years, if the interest rate is \(i\).

16. SUM TOTAL FROM A SINGLE INVESTMENT

Finds the sum total from a single investment when invested at an annual interest rate \(i\), for \(N\) years compounded \(M\) times/year.

17. PERIODIC INVESTMENT

Finds the periodic investment \(R\) required to yield sum \(S\) if \(R\) is invested at interest rate \(i\), \(M\) times/year for \(N\) years.

18. SUM FROM PERIODIC INVESTMENT

Finds the sum resulting from an amount invested at a given interest rate, \(M\) times/year for \(N\) years.

19. DEPRECIATION CHARGE (DECLINING BALANCES)

Finds the depreciation charge in a given year for any given declining balance depreciation rate.

20. DECLINING BALANCE DEPRECIATION RATE

Finds the declining balance depreciation rate such that the book value of an item equals the estimated salvage value at the end of an \(N\) year life.
21. SALVAGE VALUE

Finds the book salvage value at the end of (N) years given the declining balance depreciation rate.

22. AVERAGE GROWTH RATE AND PROJECTED SALES

Computes the "average" growth rate in sales, earnings, etc. of a company over N years. Given the total sales S_0, S_1, ..., S_n of a company for each year over a period of N years, we perform a linear regression to determine the "average" growth rate, G. Using G, we can project what the sales of the company should be in a future year.

GLBR 22B
PLOT ROUTINE

23. PLOT

Plots a function of X, f(x), on a set of coordinate axes.

24. MULTI- PLOT

Plots from 1 to 9 functions of X on the same set of coordinate axes.

25. POLAR PLOT

Plots a function, f(θ), in polar coordinates.

26. T- PLOT

Plots a set of (X, Y) data points on a set of coordinate axes.

27. HISTOGRAM

Prints a histogram of a set of numbers.

GLBR 22B
UTILITY

28. UTILITY

Contains sort, pack, and unpack subroutines.
29. ARTILLERY
   Game to determine your rank in the army artillery.

30. CRAPS
   Simulates the game of craps.

31. TIC-TAC-TOE
   Plays Tic-Tac-Toe with the user.

32. ONE ARMED BANDIT
   Simulates a "one-armed-bandit" machine.

33. BLACK JACK
   Simulates the game of "Black-Jack"

GLBR 22B
GENERAL SCIENCE

34. MASS OF NITROGEN IN CONTAINMENT SYSTEM
   Calculates the mass of nitrogen in the reactor containment system on a periodic basis to provide a measure of the leakage rate.

35. PERCENT ABSORPTION TO CONCENTRATION
   Computes the concentration of unknowns given the percent absorption read from an atomic absorption spectrophotometer.